

Let  $G = \langle g \rangle$  be cyclic of order  $n > 1$ .

(a)  $|G \times G| = |G| \times |G| = n \times n = n^2$ .

(b) If  $(a, b) \in G \times G$  then  $a = g^i$  and  $b = g^j$   
for some  $0 \leq i, j \leq n$ . Hence

$$\begin{aligned}(a, b)^n &= ((g^i)^n, (g^j)^n) = ((g^n)^i, (g^n)^j) \\ &= (e^i, e^j) \\ &= (e, e) \text{ as } g \text{ has order } n.\end{aligned}$$

(c) From (b), every element of  $G \times G$  has order at most  $n$ . Since  $n > 1$ , it follows that  $n^2 > n$ , so no element has order  $n^2$ . From this, we see that  $G \times G$  cannot be cyclic, as it doesn't contain an element of order  $n^2$  (the order of the group), which would contradict Proposition 4.14.