7(a) Suppose that $ghg = gkg$. Then, multiplying on the left by g^{-1} gives

$$g^{-1}ghg = g^{-1}gkg, \text{ so } hg = kg.$$

Then, multiplying on the right by g^{-1} gives

$$hgg^{-1} = kgg^{-1}, \text{ so } h = k.$$

(This is an application of the cancellation laws.)

(b) Suppose that $hgh = kgk$. Then $hhg = kkg$ as G is abelian.

Hence $h^2 g = k^2 g$

So $h^2 g g^{-1} = k^2 g g^{-1}$

So $h^2 = k^2$, as required.

(c) If $g = s_1$, $h = e$ & $k = r_1$ in D_3, then

$$hgh = es_1e = s_1$$

and $kgk = r_1 s_1 r_1 = s_2 r_1 = s_1 = hgh$,

but $h^2 = e$ and $k^2 = r_2 \neq e$.

(There are plenty of other options, e.g.

$g = r_1$, $h = r_2$, $k = s_1$

etc.)

8. Let $a, b \in G$. Then $ab = (ab)^{-1} = b^{-1}a^{-1} = ba$. Thus G is abelian.