(a) If $A = \{a_1, a_2, a_3, \dots\}$ & $B = \{b_1, b_2, b_3, \dots\}$ are countable, then the presentation

$$A \cup B = \{a_1, b_1, a_2, b_2, a_3, b_3, \dots\}$$

(where we delete repetitions if they occur) shows that $A \cup B$ is countable.

(b) Suppose that I is countable, for a contradiction.

From the notes, we know that $\mathbb{Q}$ is countable.

$\therefore$ using part (a), $\mathbb{Q} \cup I$ is countable.

But $\mathbb{Q} \cup I = \mathbb{R}$ which is uncountable.

Thus we have a contradiction,

and I must be uncountable.