

(a) For injectivity, let  $a_1 = (x_1, y_1)$ ,  $a_2 = (x_2, y_2) \in \mathbb{R}^2$

& suppose  $f(a_1) = f(a_2)$ .

Then  $(x_1 + y_1, x_1 - y_1) = (x_2 + y_2, x_2 - y_2)$ , so

$$\begin{aligned} x_1 + y_1 &= x_2 + y_2 && \text{--- (1)} \\ \& \ x_1 - y_1 &= x_2 - y_2 && \text{--- (2)} \end{aligned}$$

(1) + (2) gives  $2x_1 = 2x_2$ , so  $x_1 = x_2$ .

Then, from (1),  $x_1 + y_1 = x_1 + y_2$ , so  $y_1 = y_2$ .

Hence  $a_1 = a_2$  and  $f$  is injective.

For surjectivity, take any  $b = (u, v) \in \mathbb{R}^2$  and put  $a = \left(\frac{u+v}{2}, \frac{u-v}{2}\right)$ . Then  $a \in \mathbb{R}^2$  and

$$\begin{aligned} f(a) &= \left(\frac{u+v}{2} + \frac{u-v}{2}, \frac{u+v}{2} - \frac{u-v}{2}\right) \\ &= (u, v) \\ &= b. \end{aligned}$$

$\therefore f$  is surjective.

Thus, since  $f$  is injective & surjective it is bijective, with inverse  $g: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $g(u, v) = \left(\frac{u+v}{2}, \frac{u-v}{2}\right)$ .

(b) Since  $h(0, 1) = h(0, 2) = (0, 0)$  it follows that  $h$  is not injective.

Also, if  $(x, y) \in \mathbb{R}^2$  with  $h(x, y) = (0, 1)$  then  $(2x, xy) = (0, 1)$ ,

so  $x = 0$  &  $xy = 1$ , a contradiction.

$\therefore h$  is not surjective.