

(a) Using the rot/ref formulae,

$$\begin{aligned}\text{rot}_\alpha \text{ref}_\alpha \text{rot}_{-\alpha} &= \text{rot}_\alpha \text{ref}_{\alpha - (-\alpha)} \\ &= \text{rot}_\alpha \text{ref}_{2\alpha} \\ &= \text{ref}_{\alpha + 2\alpha} \\ &= \text{ref}_{3\alpha}.\end{aligned}$$

(b) Suppose that $\text{rot}_\alpha \text{ref}_\alpha \text{rot}_{-\alpha} = \text{ref}_\alpha$. Then, from part (a), $\text{ref}_{3\alpha} = \text{ref}_\alpha$. Hence 3α and α must differ by a multiple of 2π , i.e. $3\alpha = \alpha + 2n\pi$ for some $n \in \mathbb{Z}$. Hence $\alpha = n\pi$ for some $n \in \mathbb{Z}$, as required.

Now suppose that $\alpha = n\pi$ for some $n \in \mathbb{Z}$. Then, using (a),

$$\text{rot}_\alpha \text{ref}_\alpha \text{rot}_{-\alpha} = \text{ref}_{3\alpha} = \text{ref}_{3n\pi} = \text{ref}_{2n\pi + n\pi} = \text{ref}_{n\pi} = \text{ref}_\alpha,$$

as required.